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STABILITY OF TRANSONIC TWO-PHASE FLOW

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The nature of a singular point in the stability of one-dimensional transonic flow of a vapor–drop mixture in a channel of variable cross section is considered within the framework of a two-liquid hydrodynamical model. It is shown that the singular point in the case of any lags of the drops preserves the nature of a saddle inherent to homogeneous gas flow, shifting only towards the divergent part of the channel if the content of condensed phase is not too high. Here the transition of subsonic two-phase flow into supersonic flow is stable and the predominance of drop agglomeration over fragmentation and the positive curvature of the channel profile are stabilizing factors. The saddle nature of the singularity is possible only if the lag of the drops is not too high in the case of flows with a higher content of condensed phase. In the opposite case, the point at which the speed of sound is attained loses the nature of a saddle point.

A physical model and closed system of equations for the hydrodynamics of a coarse-dispersion vapor–drop mixture, taking into account the effects of relative motion and heat and mass transfer between the phases, and including seven first-order quasilinear differential equations (conservation equations) and ten final equations (four equations of state, four transfer equations, and two closure equations) has been proposed [1, 2].

It was proved that all the characteristic velocities of this type of one-dimensional nonsteady flow of a two-phase medium are real, and that the system of equations of one-dimensional nonsteady flow satisfies evolution conditions, and correctly states the problem with the initial data. From this point of view, the model of a two-phase medium can be considered physically justified.

Two of the six different characteristic velocities may change sign, passing through zero. The existence of vanishing velocity characteristic of one-dimensional nonsteady flow is due to the occurrence of singular points for the system of equations of the corresponding steady flow [3]. Flow in the neighborhood of a singular

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 163-171, January-February, 1976. Original article submitted November 23, 1974.

point is said to be transonic in analogy with ordinary gasdynamics, though the physical nature of this singularity may be entirely different ("pseudosonic"). The nature of a singular point and the stability of transonic two-phase flow is not only of theoretical interest, but also has important applications; for example, in calculating quasi-one-dimensional flow. In fact, the exact integral curve is replaced by an approximation in the neighborhood of the singular point in the course of numerical integration of the flow equations and the resulting error may be considered as a corresponding disturbance. It is safe to say that, since this disturbance is small, we may guarantee that the deviation of the disturbed solution (i.e., the result of numerical integration) will also be small from the exact solution as it is continued beyond the neighborhood of the singularity (outside this neighborhood this guarantee is supplied by the well-formedness of the problem with the initial data for the given system of equations).

We will briefly set forth the scheme of a method developed in [3] for studying the stability of arbitrary steady flow in the neighborhood of characteristic surfaces, as applied to the flow of a nonequilibrium two-phase mixture of this type in a channel of variable cross section. The system of equations given in [2] and solved for the derivatives, is written in the form

$$w_1' = \frac{Q_{10}(\varkappa-1)\frac{p}{\rho_2^0} - Q_{11}\left(w_2^2 - \frac{\rho_1^0}{\rho_1}\frac{p}{\rho_2^0}\right)}{\rho_1\left(w_1^2 - \varkappa\frac{p}{\rho_1^0}\right)\left(w_2^2 - \frac{\rho_1^0}{\rho_1}\frac{p}{\rho_2^0}\right)}; \tag{1}$$

$$w_2' = \frac{Q_{11}}{(\varkappa-1)\rho_2\frac{p}{\rho_2^0}} + \frac{\rho_1\left(w_1^2 - \varkappa\frac{p}{\rho_1^0}\right)}{(\varkappa-1)\rho_2\frac{p}{\rho_2^0}}; \tag{2}$$

$$\rho_1' = \frac{Q_1}{w_1} - \frac{\rho_1}{w_1}w_1'; \tag{3}$$

$$\rho_2' = \frac{Q_4}{w_2} - \frac{\rho_2}{w_2}w_2'; \tag{4}$$

$$u_1' = \frac{Q_3}{\rho_1 w_1} - \frac{p}{\rho_1^0 w_1}w_1' - \frac{\rho_2}{\rho_2^0}\frac{p}{\rho_1 w_1}w_2'; \tag{5}$$

$$u_2' = \frac{Q_6}{\rho_2 w_2}; \tag{6}$$

$$n' = \frac{Q_7}{w_2} - \frac{n}{w_2}w_2', \tag{7}$$

where

$$Q_{10} = w_2Q_5 - (\rho_1^0/\rho_1)(\rho_2/\rho_2^0)w_2Q_2 - (\rho_1^0/\rho_1)(p/\rho_2^0)Q_4; \tag{8}$$

$$Q_{11} = (\varkappa-1)u_1Q_1 - W_1Q_2 + (\varkappa-1)Q_3; \tag{9}$$

and Q_i $(i = 1, \ldots, 7)$ are the source terms, which depend on the form of the transfer equations and the drop fragmentation (agglomeration) rate.

Equations (1)-(7) imply that in a system of quasi-one-dimensional steady flow equations solved for the derivatives,

$$\chi_1' = F_i(\chi_m, y, y')\begin{matrix} i = 1, \ldots, 7 \\ m = 1, \ldots, 17, \end{matrix}$$

a singularity is basically found only in the equation

$$w_1' = \Omega/\varphi\psi,$$

where

$$\Omega = [(\varkappa-1)/\rho_1](p/\rho_2^0)Q_{10} - (\psi/\rho_1)Q_{11}; \tag{10}$$

$$\varphi = w_1^2 = \varkappa p/\rho_1^0; \tag{11}$$

$$\psi = w_2^2 - (\rho_1^0/\rho_1)\, p/\rho_2^0, \tag{12}$$

the other derivatives being expressed in terms of w'_1 in the usual way.

Leaving aside the question as to the nature of the characteristic velocity $\xi_1 = w_2 - [(\rho_1^0/\rho_1)\, p/\rho_2^0]^{1/2}$ let us turn to flow in a neighborhood of the singularity $\psi = 0$ (i.e., $\xi_1 = 0$).

We may pass from the space of the physical variables $\{\chi_m, x\}$, m = 1, ..., 17, to the space $x = x(\chi_m, \Omega, \psi)$, $\rho_2 = \rho_2(\chi_m, \Omega, \psi)$, where $m \neq k$ and $\{\chi_m, \Omega, \psi\}$, $m \neq k$, $\chi_k = \rho_2$, by means of the nondegenerate substitution Ω. A continuous passage through the singular point $\psi = 0$ is possible only if Ω simultaneously vanishes. The conditions $\psi = 0$ and $\Omega = 0$, considered jointly, determine a 16-dimensional surface of singular points of quasi-one-dimensional steady flow equations in the 18-dimensional space of variables $\{\chi_m, \Omega, \psi\}$. The integral curve of this system, passing through an isolated singular point, will consequently belong to the complement (to the complete space) of the surface of singular points, i.e., it will be two-dimensional, in a small neighborhood of this point. Since the point $\Omega = 0, \psi = 0$ belongs to the integral curve, the integral curve describing continuous flow in a small neighborhood of the singular point will lie on the plane $\{\Omega, \psi\}$. This makes it possible to clarify the nature of the singularity in the usual way.

We consider x as a parameter, finding

$$d\Omega/dx = M_1(\chi_m, \chi'_m, y, y', y''), m = 1, \ldots, 17; \tag{13}$$

$$d\psi/dx = M_2(\chi_m, \chi'_m), m = 1, \ldots, 17. \tag{14}$$

We eliminate the derivatives using Eqs. (1)-(17). Since all the χ'_1 are expressed in terms of $w'_1 = \Omega/\varphi\psi$,

$$d\Omega/dx = N_1(\Omega/\varphi\psi, \chi_m, y, y', y''), \tag{15}$$

$$d\psi/dx = N_2(\Omega/\varphi\psi, \chi_m). \tag{16}$$

hence

$$d\Omega/d\psi = L_1(\Omega, \psi)/L_2(\Omega, \psi) \tag{17}$$

and the nature of the singularity is determined (as usual) by the coefficients of the linear expansion of L_1 and L_2 in terms of Ω and ψ in a neighborhood of the singular point, i.e., by the eigenvalues of the matrix $\|a_{ij}\|$, where i, j = 1, 2 and where

$$a_{11} = (\partial L_1/\partial\Omega)_0;\ a_{21} = (\partial L_2/\partial\Omega)_0;\ a_{12} = (\partial L_1/\partial\psi)_0;\ a_{22} = (\partial L_2/\partial\psi)_0.$$

This may be reformulated for the singular point $\varphi = 0$ ($\xi_2 = 0$).

Let us consider flow in the neighborhood of $\psi = 0$, bearing in mind that the fact ρ_1^0/ρ_2^0 is a small parameter and that the point $\psi = w_2^2 - (\rho_1^0/\rho_1)\, p/\rho_2^0 = 0$ lies within a neighborhood of the point at infinity ($x = -\infty$, $w_2 = 0$) at which flow weakly depends on channel profile (approximate conical drain); deviation of the two-phase mixture from equilibrium is quite low in this case and we will use linear transfer equations; the typical density ρ_1^0 of the gas phase weakly varies, and the gas phase, like the liquid phase, can be considered incompressible; fragmentation and agglomeration of the drops can be disregarded.

With these remarks in mind

$$Q_1 = j - g,\ Q_2 = f,\ Q_3 = q,$$

$$Q_4 = g - j,\ Q_5 = -f,\ Q_6 = -q,\ Q_7 = 0,$$

$$q = 2\pi k_1 n\delta(T_2 - T_1),\ f = 6\pi\eta_1 n\delta(w_2 - w_1),$$

$$j = 2\pi\rho_1^0 D_1 n\delta U_+\left[\frac{p_{eq}(T_2) - p}{p}\right],$$

$$g = 2\pi\rho_1^0 D_1 n\delta\, \frac{p_{eq}(T_1)}{p}\, U_+\left[\frac{p - p_{eq}(T_1)}{p}\right].$$

We will consider that the thermophysical properties of the gas and liquid are constant, so that in a neighborhood of $\psi = 0$,

$$\frac{d\Omega}{dx} = \frac{d\rho_2}{dx}\,[\Omega_j j_{\rho_2} + \Omega_f f_{\rho_2}] + \frac{dw_1}{dx}\,[\Omega_{w_1} + \Omega_f f_{w_1}] \tag{18}$$

$$+\frac{dw_2}{dx}[\Omega_{w_2}+\Omega_f f_{w_2}]+\frac{dp}{dx}[\Omega_p+\Omega_j j_p]+\frac{d\rho_1}{dx}\Omega_{\rho_1}+\frac{du_2}{dx}\Omega_j j_{u_2}+\frac{4(\varkappa-1)}{x^2}\frac{\rho_2}{\rho_2^0}\frac{p}{\rho_1}w_2,$$

where (the corresponding pressure derivatives are given subscripts)

$$\Omega_j=\frac{\varkappa-1}{\rho_1}\frac{p}{\rho_2^0}w_2(w_2-w_1);\ \Omega_f=-\frac{\varkappa-1}{\rho_1}\frac{p}{\rho_2^0}w_2;$$

$$\Omega_{w_2}=\frac{\varkappa-1}{\rho_1}\frac{p}{\rho_2^0}\left(2jw_2-jw_1-\frac{\rho_1^0}{\rho_1}f\right)-2\frac{w_2}{\rho_1}Q_{11};$$

$$\Omega_{w_1}=-\frac{\varkappa-1}{\rho_1}\frac{p}{\rho_2^0}jw_2;\ \Omega_p=\frac{\rho_1^0}{\rho_1^2}\frac{1}{\rho_2^0}Q_{11}+\frac{\varkappa-1}{\rho_1}\frac{1}{\rho_2^0}w_2Q_5;$$

$$\Omega_{\rho_1}=-\frac{\rho_1^0}{\rho_1^3}\frac{p}{\rho_2^0}Q_{11}-\frac{\varkappa-1}{\rho_1^2}\frac{p}{\rho_2^0}Q_{10};$$

$$j_{\rho_2}=\frac{1}{3}\frac{j}{\rho_2};\ j_p=\frac{j}{p}\frac{p_{eq2}}{p_{eq2}-p};\ j_{u_2}=\frac{r}{\varkappa-1}\frac{j}{u_1u_2}\frac{p_{eq2}}{p_{eq2}-p}\frac{T_1}{T_2};$$

$$f_{\rho_2}=\frac{1}{3}\frac{f}{\rho_2};\ f_{w_2}=\frac{f}{w_2-w_1};\ f_{w_1}=-\frac{f}{w_2-w_1}.$$

We eliminate the derivatives from Eqs. (18) and, disregarding terms of the order of ρ_2/ρ_2^0, we obtain

$$\frac{d\Omega}{dx}=\frac{\Omega}{\psi}\left[-\frac{2}{\varkappa-1}\frac{\rho_2^0}{\rho_2}\frac{w_2}{p}Q_{11}+\frac{1}{\rho_2}(5w_2-2w_1)\right.$$
$$\left.\times\left(j-\frac{f}{w_2-w_1}\right)\right]+\left[-\frac{2}{\varkappa-1}\frac{\rho_2^0w_2}{\rho_2p}\frac{Q_{11}^2}{\rho_1}+\frac{1}{\rho_2}(5w_2-2w_1)\left(j-\frac{f}{w_2-w_1}\right)\frac{Q_{11}^2}{\rho_1}.\right.\qquad(19)$$

We now calculate

$$\frac{d\psi}{dx}=2w_2\frac{dw_2}{dx}-\frac{\rho_1^0}{\rho_1}\frac{1}{\rho_2^0}\frac{dp}{dx}+\frac{\rho_1^0}{\rho_1^2}\frac{p}{\rho_2^0}\frac{d\rho_1}{dx}.$$

Eliminating the derivatives, we obtain ρ_2/ρ_2^0

$$\frac{d\psi}{dx}=\frac{\Omega}{\psi}\left[\frac{2}{\varkappa-1}\frac{\rho_2^0w_2}{\rho_2p}\rho_1\right]+\left[\frac{2}{\varkappa-1}\frac{\rho_2^0w_2}{\rho_2p}Q_{11}\right].\qquad(20)$$

Equations (19) and (20) imply that in the neighborhood of $\psi=0$, $\Omega=0$

$$\frac{d\Omega}{d\psi}=\frac{a_{11}\Omega+a_{12}\psi}{a_{21}\Omega+a_{22}\psi},$$

where

$$a_{11}=-\frac{2}{\varkappa-1}\frac{\rho_2^0w_2}{\rho_2p}Q_{11}+\frac{1}{\rho_2}(5w_2-2w_1)\left(j-\frac{f}{w_2-w_1}\right);$$

$$a_{12}=-\frac{2}{\varkappa-1}\frac{\rho_2^0w_2}{\rho_2p}\frac{Q_{11}^2}{\rho_1}+\frac{1}{\rho_2}(5w_2-2w_1)\left(j-\frac{f}{w_2-w_1}\right)\frac{Q_{11}}{\rho_1};$$

$$a_{21}=\frac{2}{\varkappa-1}\frac{\rho_2^0w_2}{\rho_2p}\rho_1;\quad a_{22}=\frac{2}{\varkappa-1}\frac{\rho_2^0w_2}{\rho_2p}Q_{11}.$$

Since

$$\det\|a_{ij}\|=a_{11}a_{22}-a_{21}a_{12}=0,$$

the characteristic equation

$$\begin{vmatrix}a_{11}-\lambda & a_{12}\\ a_{21} & a_{22}-\lambda\end{vmatrix}=0$$

has real roots, one of which is $\lambda_1=0$. The stability of flow in the neighborhood of $\Omega=0$, $\psi=0$ is determined by the sign of

$$\lambda_2 = (1/2)\,\mathrm{Sp}\,\|a_{ij}\| = (1/2)(a_{11} + a_{22}) = (1/2\rho_2)(5w_2 - 2w_1)[j - f/(w_2 - w_1)].$$

Since $f/(w_2-w_1) > 0$, $\lambda_2 > 0$ for $w_2 < (2/5)w_1$ in the absence of phase transformations, and the transition of two-phase flow through the front is stable, though not asymptotically stable.

Evaporation is a destabilizing factor, though a simple estimate following from $\mathrm{Sc} \approx 1$, $f/(w_2-w_1) = 6\pi n\delta\eta_1$, $j = 2\pi n\delta\eta_1 (p_{eq2}-p)/p$ shows that

$$j - f/(w_2 - w_1) > 0$$

only when $(p_{eq2}-p)/p > 3$, i.e., at a very significant deviation from equilibrium. Thus, such two phase flows are unstable in a neighborhood of the front of a pseudosonic wave (the singular point $\xi_1 = 0$ of the system of quasi-one-dimensional steady flow equations) for a slip coefficient $\nu = (w_1-w_2)/w_1 < 3/5$. This condition is obviously realized in the neighborhood of the initial point.

Let us now turn to two-phase flow in a neighborhood of the singular point $\varphi = 0\,(\xi_2 = 0)$. The thermophysical properties will be considered as before, constant, though this time we will not assume that the physical density of the gas phase ρ_1^0 and the numerical concentration of drops n are invariant (spatial homogeneity). We use the total (nonlinear) transfer equations

$$q = 2\pi k_1 n\delta(1 + (3/10)\,\mathrm{Pr}^{1/3}\,\mathrm{Re}^{1/2})(T_2 - T_1); \tag{21}$$

$$f = 6\pi\eta_1 n\delta(1 + (3/16)\,\mathrm{Re})(w_2 - w_1); \tag{22}$$

$$j = 2\pi\,\rho_0^1 D_1 n\delta\left(1 + \frac{3}{10}\,\mathrm{Sc}^{1/3}\,\mathrm{Re}^{1/2}\right)U_+\left[\frac{p_{eq}(T_2) - p}{p}\right]; \tag{23}$$

$$g = 2\pi\rho_1^0 D_1 n\delta\left(1 + \frac{3}{10}\mathrm{Sc}^{1/3}\mathrm{Re}^{1/2}\right)\frac{p_{eq}(T_1)}{p}\,U_+\left[\frac{p - p_{eq}(T_1)}{p}\right]. \tag{24}$$

We also assume that in transonic two-phase flow,

$$w_2^2 \gg \frac{\rho_1^0}{\rho_2^0}\frac{p}{\rho_1} \quad \text{and} \quad \psi \approx w_2^2.$$

In the general case, i.e., taking into account all irreversible processes occurring in a two-phase mixture, the equation for the expansion coefficients $d\Omega/dx$ in a neighborhood of the acoustic surface turned out to be highly cumbersome. Even a rather simple estimate, however, demonstrates that kinematic nonequilibrium (particle lag, if we are speaking of accelerated flow) will play the most important role in flows of a coarse-dispersion two-phase medium. Assuming, for example, that (apparently, entirely reasonable constraints)

$$\mathrm{Re} \geqslant 1,\ (p_{eq2} - p)/p \leqslant 1/2,\ (p - p_{eq1})/p \leqslant 1/2,$$

we find from the transfer equations (21)-(24) that we have adopted that

$$j \leqslant 0.18f/(w_2 - w_1),\ g \leqslant 0.09f/(w_2 - w_1),$$

i.e., confirmation of the comparatively secondary role mass transfer plays in these flows.

On the other hand, if we assume that $(T_2-T_1)/T_1 \leq 1/2$, we find from $\mathrm{Pr} = \eta_1 c_{1p}/k_1 \approx 1$ the estimate

$$q = \pi n\delta k_1\frac{10 + 3\mathrm{Re}^{1/2}}{5}(T_2 - T_1) \leqslant \pi n\delta\eta_1\frac{10 + 3\mathrm{Re}^{1/2}}{10}\frac{\varkappa}{\varkappa - 1}\frac{R}{M}T_1 = \pi n\delta\eta_1\frac{10 + 3\mathrm{Re}}{10}\frac{1}{\varkappa - 1}w_1^2.$$

Thus, resorting to Eq. (24), we find that

$$q \leqslant 0.54f(w_2 - w_1).$$

Thus, within the defined boundaries we have grounds for taking into account at first only the kinematic lag of the drops, as is often carried out in the literature. Flow stability in the neighborhood of the singular point $\varphi = 0$, $\Omega = 0$ is solved, as was proved above, for the singular point $\psi = 0$, $\Omega = 0$ by the form of the coefficients a_{ij}(i, j = 1, 2) of bilinear form $d\Omega/d\varphi = (a_{11}\Omega + a_{12}\varphi)/(a_{21}\Omega + a_{22}\varphi)$. In this case, taking into account only kinematic nonequilibrium,

$$a_{11} = f\left[-\frac{1}{\rho_1}(\varkappa - 1)(1 - \omega)\left(\omega\frac{w_1 - w_2}{w_1} + \frac{w_2}{w_1}\frac{3\mathrm{Re}}{16 + 3\mathrm{Re}}\right) - \frac{1}{\rho_1}\frac{w_1}{w_1 - w_2}\left(1 + \frac{w_1 - w_2}{w_1} + \frac{w_2}{w_1}\frac{3\mathrm{Re}}{16 + 6\mathrm{Re}}\right)\right]; \tag{25}$$

$$a_{12} = f^2 \left\{ \frac{w_1 - w_2}{\rho_1\rho_2} (\varkappa - 1)(1 - \omega) \left[1 + \frac{w_2}{w_1 - w_2} \frac{16 + 6\mathrm{Re}}{48 + 9\mathrm{Re}} \right] \right.$$
$$+ \frac{w_1}{\rho_1\rho_2} \left[1 + \frac{w_2}{w_1 - w_2} \frac{16 + 6\mathrm{Re}}{48 + 9\mathrm{Re}} - (\varkappa - 1)(1 - \omega)\,\omega \right]$$
$$\left. + \frac{w_2}{\rho_1\rho_2} (\varkappa - 1)(1 - \omega^2) \right\} - f \frac{y'}{y} \left\{ 2 \frac{w_2^2}{\rho_1} (\varkappa - 1)(1 - \omega) \right.$$
$$\times (w_2 - w_1) \left[\frac{16 + 6\mathrm{Re}}{16 + 3\mathrm{Re}} + 2\omega + \frac{1}{2(\varkappa - 1)} \right]$$
$$\left. + 2 \frac{w_2^2}{\rho_1} w_1 \frac{16 + 6\mathrm{Re}}{16 + 3\mathrm{Re}} \right\} + \frac{1}{w_2} \frac{32 + 3\mathrm{Re}}{48 + 9\mathrm{Re}} f\Phi + \frac{y''}{y} 2 w_2^2 u_1 w_1 .$$

Substituting $d\varphi/dx$ in the neighborhood $\varphi = 0$, $\Omega = 0$, we find that

$$d\varphi/dx = (\Omega/\varphi) \left[\frac{(\varkappa + 1) w_1}{w_2^2} \right] + (w_1/\rho_1) Q_1 - (\varkappa/\rho_1) Q_2 .$$

Thus

$$a_{21} = [(\varkappa + 1) w_1]/w_2^2,$$
$$a_{22} = \frac{1}{\rho_1} \left\{ j w_1 + g [w_2 - (\varkappa + 1) w_1] - \varkappa f - 2\rho_1 w_1^2 \frac{y'}{y} \right\}$$

or, taking into account only velocity relaxation,

$$a_{22} = -(\varkappa/\rho_1) f - 2 w_2^2 y'/y .$$

The sufficient stability condition for the solution in the neighborhood of the singular point $\varphi = 0$, $\Omega = 0$, where

$$d\Omega/d\varphi = (a_{11}\Omega + a_{12}\varphi)/(a_{21}\Omega + a_{22}\varphi),$$

is given by

$$a_{11} + a_{22} < 0.$$

A singular point may also be a stable node, stable focus, and, finally, a saddle point, such that, as was proved in [3], the integral curve passing through the singularity in the natural positive direction will be a stable solution. However, in the case of accelerated flow with particle lag $f < 0$ and, consequently, $a_{11} > 0$ (25). The sufficient stability condition can therefore hold only when $a_{22} < 0$ (when $y' > 0$), i.e., stable transition of two-phase nonequilibrium flow with condensed particle lag beyond the speed of sound is possible only in a divergent channel.

Using equations for the expansion coefficients, we find that

$$a_{11}a_{22} - a_{12}a_{21} = f^2 \left\{ \frac{1}{\rho_1^2} \varkappa (\varkappa - 1)(1 - \omega) \left[\omega \frac{w_1 - w_2}{w_1} \right. \right.$$
$$\left. + \frac{w_2}{w_1} \frac{3\mathrm{Re}}{16 + 3\mathrm{Re}} \right] + \frac{1}{\rho_1^2} \varkappa \frac{w_1}{w_1 - w_2} \left[1 + \frac{w_1 - w_2}{w_1} + \frac{w_2}{w_1} \frac{3\mathrm{Re}}{16 + 3\mathrm{Re}} \right]$$
$$- \frac{1}{\rho_1\rho_2} \frac{w_1}{w_2} \frac{w_1 - w_2}{w_2} (\varkappa^2 - 1)(1 - \omega) \left[1 + \frac{w_2}{w_1 - w_2} \frac{16 + 6\mathrm{Re}}{48 + 9\mathrm{Re}} \right]$$
$$- \frac{1}{\rho_1\rho_2} \left(\frac{w_1}{w_2} \right)^2 (\varkappa + 1) \left[1 + \frac{w_2}{w_1 - w_2} \frac{16 + 6\mathrm{Re}}{48 + 9\mathrm{Re}} - (\varkappa - 1)(1 - \omega)\,\omega \right]$$
$$\left. - \frac{1}{\rho_1\rho_2} \frac{w_1}{w_2} (\varkappa^2 - 1)(1 - \omega^2) \right\} + f \frac{y'}{y} \left\{ \frac{w_1^2}{\rho_1} 2 (\varkappa - 1)(1 - \omega) \left[\omega \frac{w_1 - w_2}{w_1} \right. \right.$$
$$\left. + \frac{w_2}{w_1} \frac{3\mathrm{Re}}{16 + 3\mathrm{Re}} \right] + 2 \frac{w_1^2}{\rho_1} \frac{w_1}{w_1 - w_2} \left[1 + \frac{w_1 - w_2}{w_1} + \frac{w_2}{w_1} \frac{3\mathrm{Re}}{16 + 3\mathrm{Re}} \right]$$
$$+ 2 \frac{w_1 (w_1 - w_2)}{\rho_1} (\varkappa^2 - 1) \left[\frac{16 + 6\mathrm{Re}}{16 + 3\mathrm{Re}} + 2\omega + \frac{1}{2(\varkappa - 1)} \right]$$
$$\left. + 2 \frac{w_1^2}{\rho_1} (\varkappa + 1) \frac{16 + 6\mathrm{Re}}{16 + 3\mathrm{Re}} \right\} - (\varkappa + 1) \frac{w_1}{\rho_1} [(\varkappa - 1)(1 - \omega)(w_1 - w_2) + w_1]$$
$$\times \frac{32 + 3\mathrm{Re}}{48 + 9\mathrm{Re}} f\Phi - \frac{y''}{y} 2 (\varkappa + 1) u_1 w_1^2 ,$$

where $\Phi = K_B \exp(-1/W)$ (relative drop production).

An analysis of the coefficient of f^2 shows that it is negative for any lags of the condensed phase (any relative slips $\rho_1/\rho_2 > 3\varkappa/(\varkappa+1)$ ($\beta < 0.419$, in the case $\varkappa = 5/4$). Here the singularity realized in a divergent channel is obviously a saddle-point if we do not bear in mind drop production nor the influence of the curvature of the channel profile. Drop agglomeration ($\Phi < 0$), which predominates over fragmentation, and the positive curvature of the channel profile ($y'' > 0$) only strengthen this conclusion. The saddle-point nature of the singularity at higher contents of condensed phase predominating over drop fragmentation in the negative curvature of the profile (i.e., the same as in the case of a pure gas) is possible only for slips of not too high a magnitude. In the opposite case det $\| a_{ij} \| > 0$ and the nature of the singularity differs. The transition point beyond the speed of sound loses the nature of a saddle point.

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USE OF THE PARAMETRIX METHOD FOR ESTIMATING EFFECTIVE ELASTIC MODULI OR RANDOMLY NONHOMOGENEOUS ELASTIC BODIES

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UDC 539.31

The magnitude of the elasticity tensor of a comparison body remains unclarified if we use a singular approximation [1] to estimate the effective values of the elasticity tensor. Below we will use a parametrix method [2] to determine the first approximation of the random component of the deformation tensor and the effective values of the elasticity tensor, and will also compare the exact solution for one particular heterogeneous and a previously used approximation.

The effective value of the elasticity tensor $\boldsymbol{\lambda}^0$ is determined by

$$\boldsymbol{\lambda}^0 \langle \boldsymbol{\varepsilon} \rangle = \langle \boldsymbol{\lambda} \rangle \langle \boldsymbol{\varepsilon} \rangle + \langle \boldsymbol{\lambda}' \boldsymbol{\varepsilon}' \rangle,$$

where $\boldsymbol{\lambda}' = \boldsymbol{\lambda} - \langle \lambda \rangle$; $\boldsymbol{\varepsilon}' = \boldsymbol{\varepsilon} - \langle \boldsymbol{\varepsilon} \rangle$, and the stress tensor satisfies the equilibrium equation

$$\nabla(\boldsymbol{\lambda\varepsilon}) = 0.$$

The solution of the equation will be found in the form of a space potential

$$\boldsymbol{\varepsilon}'' = \int \operatorname{def}_x \mathbf{G}(\mathbf{x}, \mathbf{y}) \mathbf{f}(\mathbf{y}) \, dv_y, \tag{1}$$

where $\operatorname{def}_x = (1/2)[\nabla x + (\nabla x)^T]$; and $\mathbf{G}(\mathbf{x}, \mathbf{y})$ is the parametrix [4] of the equilibrium equation, which coincides with the "principal" polar part of Green's tensor of a heterogeneous and isotropic medium.

We assume that $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^0 + \boldsymbol{\varepsilon}''$, $\boldsymbol{\varepsilon}^0 = \text{const}$, and substituting Eq. (1) in the equilibrium equation, we obtain the integral equation

Tallin. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 171-175, January-February, 1976. Original article submitted March 18, 1975.